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ABSTRACT

A computer program for simultaneously factor analyzing dispersion matrices obtained from independent groups is described. This program is useful when a battery of tests has been administered to samples of examinees from several populations and one wants to study similarities and differences in factor structure between the different populations. (CK)

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SIFASP

A GENERAL COMPUTER PROGRAM FOR SIMULTANEOUS FACTOR ANALYSIS IN SEVERAL POPULATIONS

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SIFASP

A General Computer Program for Simultaneous Factor Analysis in Several Populations

1. Introduction

1.1 The General Model

We shall describe a computer program for simultaneously factor analyzing dispersion matrices obtained from independent groups. A common situation, when this program will be useful, is when a battery of tests has been administered to samples of examinees from several populations and one wants to study similarities and differences in factor structures between the different populations. The most important feature of the program is that parameters in the factor analysis models (factor loadings, factor variances, factor covariances, and unique variances) for the different populations may be assumed to be known a priori or specified to be invariant over populations. Given such a specification, the model is estimated by the maximum likelihood method yielding a large sample χ^2 test of goodness of fit. By computing several solutions under different specifications one can test various hypotheses. For example one can test the hypothesis of an invariant factor pattern. The method is capable of dealing with any degree of invariance, from the one extreme, where nothing is invariant, to the other extreme, where everything is invariant. A detailed account of the method, on which the program is based, is given by Jöreskog (1970).

Consider a set of m populations. These may be different nations, or culturally different groups, groups of individuals selected on the basis of

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some known or unknown selection variable, groups receiving different treatments, etc. In fact, they may be any set of exclusive groups of individuals that are clearly defined. It is assumed that a battery of p tests has been administered to a sample of individuals from each population. The battery of tests need not be the same for each group, but to be interesting, it is necessary that some of the tests in each battery are the same or at least content-wise equivalent.

Let x_g be a vector of order p, representing the measurements obtained in group g. We regard x_g as a random vector with mean vector μ_g and variance-covariance Σ_g. It is assumed that a factor analysis model holds in each population so that x_g can be accounted for by k common factors f_g and p unique factors z_g, as

$$(1) \qquad x_g = \mu_g + \Lambda_g f_g + z_g \quad ,$$

with $\mathcal{E}(f_g) = 0$ and $\mathcal{E}(z_g) = 0$ and Λ_g a factor pattern of order $p_g \times k_g$. The usual factor analytic assumptions then imply that

$$(2) \qquad \Sigma_g = \Lambda_g \Phi_g \Lambda_g' + \psi_g^2$$

where Φ_g is the variance-covariance matrix of f_g and ψ_g^2 is the diagonal variance-covariance matrix of z_g.

In addition to assuming that a factor analytic model holds in each population the model may specify that certain parameters in Λ_g, Φ_g, ψ_g, $g = 1,2,\ldots,m$ have assigned values and that some set of unknown elements in Λ_g, Φ_g and ψ_g are the same for all g. Thus, parameters in Λ_g, Φ_g and ψ_g, $g = 1,2,\ldots,m$ are of three kinds: (i) fixed parameters which

have been assigned given values, (ii) constrained parameters which are unknown but equal to one or more other parameters and (iii) free parameters which are unknown and not constrained to be equal to any other parameter. Equality constraints between parameters for the same populations may also be used though this would be unusual in practice. The advantage of this approach is the great generality and flexibility obtained by the various specifications that may be imposed. The most common situation is when the same battery has been administered to each group and when the whole factor pattern Λ_g is assumed to be invariant over groups. This case will hereafter be referred to as the standard case.

1.2 Identification of Parameters

Before an attempt is made to estimate a model of this kind, the identification problem must be examined. The identification problem depends on the specification of fixed, free and constrained parameters. Under a given specification, each Λ_g, Φ_g and ψ_g generates one and only one Σ_g but it is well known that ... Λ_g and Φ_g generate the same Σ_g. It should be noted that if Λ_g is replaced by $\Lambda_g T_g^{-1}$ and Φ_g by $T_g \Phi_g T_g'$, where T_g is an arbitrary nonsingular matrix of order $k_g \times k_g$, then Σ_g is unchanged. Since T_g has k_g^2 independent elements, this suggests that k_g^2 independent conditions should be imposed on Λ_g and/or Φ_g to make these uniquely defined and hence that $\sum_{g=1}^{m} k_g^2$ independent conditions altogether should be imposed. However, when equality constraints over groups are taken into account, all the elements of all the transformation matrices are not independent of each other and therefore a lesser

number of conditions need to be imposed. It is hard to give further specific rules in the general case. To make sure that all indeterminacies have been eliminated, one should verify that the only transformations $T_1, T_2, \ldots, T_m$ that preserve the specification about fixed, free and constrained parameters are identity matrices.

In the standard case when the whole factor pattern is invariant over groups, however, a more precise consideration of the identification problem can be given. Suppose that the Λ is replaced by $\Lambda^* = \Lambda T^{-1}$ and each Φ_g is replaced by $\Phi^*_g = T\Phi_g T'$, $g = 1,2,\ldots,m$, where T is an arbitrary nonsingular matrix of order $k \times k$. Then each Σ_g remains the same. Since the matrix T has k^2 independent elements, this means that at least k^2 independent conditions must be imposed on the parameters in Λ , $\Phi_1, \Phi_2, \ldots, \Phi_m$ to make these uniquely defined.

The most convenient way of doing this is to let all the Φ_g be free and to fix one nonzero element and at least $k - 1$ zeros in each column of Λ . In an exploratory study one can fix exactly $k - 1$ zeros in almost arbitrary positions. For example one may choose zero loadings where one thinks there should be "small" loadings in the factor pattern. The resulting solution may be rotated further, if desired, to facilitate better interpretation. In a confirmatory study, on the other hand, the positions of the fixed zeros, which often exceed $k - 1$ in each column, are given *a priori* by an hypothesis and the resulting solution cannot be rotated without destroying the fixed zeros.

1.3 Estimation and Testing of the Model

Let N_g be the number of individuals in the sample from the g^{th} population and let $\bar{x}_g$ be the usual sample mean vector and S_g the usual sample variance-covariance matrix with $n_g = N_g - 1$ degrees of freedom. The only requirement for the sampling procedure is that it produces independent measurements for the different groups.

If we assume that x_g has a multinormal distribution it follows that S_g has a Wishart distribution based on Σ_g and n_g degrees of freedom. The logarithm of the likelihood for the g^{th} sample is

(3) $$\log L_g = -\frac{1}{2} n_g[\log|\Sigma_g| + tr(S_g\Sigma_g^{-1})] \quad .$$

Since the samples are independent, the log-likelihood for all the samples

(4) $$\log L = \sum_{g=1}^{m} \log L_g \quad .$$

Maximum likelihood estimates of the unknown elements in Λ_g, Φ_g, Ψ_g, $g = 1,2,\ldots,m$, may be obtained by maximizing $\log L$. However, it is slightly more convenient to minimize

(5) $$F = \frac{1}{2} \sum_{g=1}^{m} n_g[\log|\Sigma_g| + tr(S_g\Sigma_g^{-1}) - \log|S_g| - p]$$

instead. At the minimum, F equals minus the logarithm of the likelihood ratio for testing the hypothesis implied by the model against the general alternative that each Σ_g is unconstrained. Therefore, twice the minimum value of F is approximately distributed, in large samples, as χ^2 with degrees of freedom equal to

(6) $$d = \frac{1}{2} p(p + 1) - t$$

where t is the total number of independent parameters estimated in the model.

The minimization of F with respect to the independent parameters is done by means of a modification of the iterative method of Fletcher and Powell (1963) described by Gruvaeus and Jöreskog (1970). The minimization method makes use of the first-order derivatives and approximations to the second-order derivatives of F and converges rapidly from an arbitrary starting point to a local minimum of F. If there are several minima of F there is no guarantee that the method will converge to the absolute minimum.

The adaptation of the problem of minimizing F to the Fletcher-Powell method is described by Jöreskog (1970, section 2.4).

1.4 Scaling of Observed Variables

When the units of measurements in the different tests are arbitrary, it is usually convenient, though not necessary, to rescale the observed variables, before the factor analysis. Let

(7) $$S = (1/n) \sum_{g=1}^{m} n_g S_g \quad ,$$

with $n = \sum_{g=1}^{m} n_g$ and let

(8) $$D = (\text{diag } S)^{-1/2} \quad .$$

Then the variance-covariance matrices for the rescaled variables are

(9) $$S_g^* = DS_gD \quad , \qquad g = 1,2,\ldots,m \; .$$

The weighted average of the S_g^* is a correlation matrix. The advantage of this rescaling is that, when combined with the rescaling of the factors as described in the next section, the factor loadings are of the same order of magnitude as usual when correlation matrices are analyzed and when factors are standardized to unit variances. This makes it easier to choose start values for the minimization (see Jöreskog, 1970, section 3.5) and interpret the results.

It should be pointed out that it is not permissible to standardize the variables in each group and to analyze the correlation matrices instead of the variance-covariance matrices. This violates the likelihood function (4) which is based on the distribution of the observed variances and covariances.

1.5 Scaling of Factors

The fixed nonzero loading in each column of Λ can have any value. This is only used to fix a scale for each factor which is common to all groups. In the standard case, when the maximum likelihood solution has been obtained, the factors may be rescaled so that their average variance is unity. This rescaling is obtained as follows. Let

(10) $$\hat{\Phi} = (1/n) \sum_{g=1}^{m} n_g \hat{\Phi}_g \quad ,$$

with $n = \sum_{g=1}^{m} n_g$, as before, and

(11) $$D = (\mathrm{diag}\ \hat{\Phi})^{-1/2} \quad .$$

Then the rescaled solution is

(12) $\hat{\Lambda}^* = \hat{\Lambda}D^{-1}$

(13) $\hat{\Phi}^*_g = D\hat{\Phi}_g D \quad , \qquad g = 1,2,\ldots,m$.

The matrix $\hat{\Lambda}^*$ has zeros wherever $\hat{\Lambda}$ has zeros but the fixed nonzeros in $\hat{\Lambda}$ have changed their values. The weighted average of $\hat{\Phi}^*_g$ is a correlation matrix.

2. The Program

In this section we describe briefly what the program does. Details about the input and output are given in sections 3 and 4 respectively.

2.1 What the Program Does

The input data may be correlation matrices with standard deviations or dispersion matrices. From these input matrices, variables may be selected to be included in the analysis, so that the matrices to be analyzed may be of smaller order than the input matrices. Variables may also be interchanged with one another. The matrices to be analyzed may be dispersion matrices or dispersion matrices scaled by the program (see 1.4).

The user can request an accurate or an approximate solution. If an accurate solution is requested, the iterations of the minimization method are continued until the minimum of the function is found, the convergence criterion being that the magnitude of all derivatives be less than .00005N, where $N = (1/m) \sum_{g=1}^{m} n_g$. The solution is then usually correct to three significant digits. If an approximate solution is requested, the iterations terminate when the decrease in function values is less than 5%. The approximate solution may be useless but the residuals and the value of χ^2 will usually give an indication of how reasonable the hypothesized model is. The option of an approximate solution has been included in the program for the purpose of saving computer time in exploratory studies where the primary purpose is to find a reasonable model. Once such a model has been found, an accurate solution may be computed.

A variety of options for the printed output is available. Residuals for each population may be printed. These are defined as the differences between observed (S_g) and estimated (Σ_g) variances and covariances, which are useful for judging the goodness of fit of the model to the data. χ^2 is printed as an overall goodness of fit test statistic and, in one version of the program, standard errors for the estimated parameters may be requested (see 2.3).

2.2 How Fixed, Free and Constrained Parameters Are Specified

The elements of the parameter matrices are ordered as follows. The matrices are assumed to be in the order $\Lambda_1, \Lambda_2, \ldots, \Lambda_m$, $\Phi_1, \Phi_2, \ldots, \Phi_m$, $\Psi_1, \Psi_2, \ldots, \Psi_m$ and within each matrix, the elements are ordered row-wise. Only the lower half including the diagonal of the symmetric matrices $\Phi_1, \Phi_2, \ldots, \Phi_m$ are stored. The diagonal matrices $\Psi_1, \Psi_2, \ldots, \Psi_m$ are treated as row-vectors.

For each of the parameter matrices, a pattern matrix is defined, with elements 0, 1, 2 and 3 depending on whether the corresponding element in the parameter matrix is fixed, free, constrained follower and constrained leader, respectively. A constrained parameter is called a constrained leader the first time it appears in the sequence. The parameters, appearing later in the sequence and assumed to be equal to the constrained leader are called constrained followers.

The above technique defines uniquely the positions of the fixed, free and constrained leader parameters. It does not define, however, which followers go with which leaders, if there is more than one leader. To do so one must specify all the followers associated with a given leader. This is done by assigning to each leader and follower a four-digit number MCCC,

where M defines the matrix in which the constrained parameter appears. $M = 1$ for Λ, 2 for Φ and 3 for Ψ, where Λ is $\Lambda_1, \Lambda_2, \ldots, \Lambda_m$ reading row-wise one matrix after the other, Φ is $\Phi_1, \Phi_2, \ldots, \Phi_m$ and Ψ is $\Psi_1, \Psi_2, \ldots, \Psi_m$. The position of the parameter in the matrix is described by CCC. For example,

1001 1005 2003

defines the first element in Λ, λ_1, to be equal to the fifth element in Λ, λ_5, as well as the third element in Φ, ϕ_3, where λ_1 is the leader and λ_5 and ϕ_3 are the followers.

Pattern matrices have to be provided for each matrix containing both fixed and free parameters and for each matrix containing constrained parameters. Patterns for matrices whose elements are all fixed or all free are set up by the program.

We give a simple example to illustrate the above specifications. Suppose we have two populations and

$$\Lambda_1 = \begin{bmatrix} \lambda_1 & 0 \\ \lambda_3 & 0 \\ 0 & \lambda_6 \\ 0 & \lambda_8 \end{bmatrix} \qquad \Phi_1 = \begin{bmatrix} 1 & \\ \phi_2 & 1 \end{bmatrix} \qquad \Psi_1 = \begin{bmatrix} \psi_1 & 0 & 0 & 0 \\ 0 & \psi_2 & 0 & 0 \\ 0 & 0 & \psi_3 & 0 \\ 0 & 0 & 0 & \psi_4 \end{bmatrix}$$

$$\Lambda_2 = \begin{bmatrix} \lambda_9 & 0 \\ \lambda_{11} & 0 \\ 0 & \lambda_{14} \\ 0 & \lambda_{16} \end{bmatrix} \qquad \Phi_2 = \begin{bmatrix} 1 & \\ \phi_5 & 1 \end{bmatrix} \qquad \Psi_2 = \begin{bmatrix} \psi_5 & 0 & 0 & 0 \\ 0 & \psi_6 & 0 & 0 \\ 0 & 0 & \psi_7 & 0 \\ 0 & 0 & 0 & \psi_8 \end{bmatrix}$$

with $\lambda_1 = \lambda_3 = \lambda_9 = \lambda_{11}$, $\lambda_{14} = \lambda_{16}$, $\psi_1 = \psi_2 = \psi_5 = \psi_6$ and $\psi_7 = \psi_8$. The pattern matrices for Λ_1 , Λ_2 , Φ_1 , Φ_2 , Ψ_1 and Ψ_2 are

$$P_{\Lambda_1} = \begin{bmatrix} 3 & 0 \\ 2 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \qquad P_{\Phi_1} = \begin{bmatrix} 0 & \\ 1 & 0 \end{bmatrix} \qquad P_{\Psi_1} = \begin{bmatrix} 3 & \text{[illegible]} & \text{[illegible]} & 1 \end{bmatrix}$$

$$P_{\Lambda_2} = \begin{bmatrix} 2 & 0 \\ 2 & 0 \\ 0 & 3 \\ 0 & 2 \end{bmatrix} \qquad P_{\Phi_2} = \begin{bmatrix} 0 & \\ 1 & 0 \end{bmatrix} \qquad P_{\Psi_2} = \begin{bmatrix} 2 & 2 & 3 & 2 \end{bmatrix}$$

and the specifications of leaders and followers are

1001	1003	1009	1011
1014	1016		
3001	3002	3005	3006
3007	3008		

In this model ten independent parameters will be estimated. This is the number of 3's and 1's in the pattern matrices.

In addition to the above specifications for fixed, free and constrained parameters, start values have to be given for all parameters, except when one or more of the parameter matrices are of standard form, i.e., $\Lambda_g = I$, $\Phi_g = I$, $\Psi_g = 0$, $g = 1,2,\ldots,m$. The start values define the fixed parameters and initial values for the minimization procedure for the other parameters. Constrained parameters which are assumed to be equal must be given the same values. Otherwise, initial values may be chosen arbitrarily but the closer they are to the final solution the less computer time it will take to reach this solution (see Jöreskog, 1970, section 3.5).

2.3 Limitations

The program is written in FORTRAN IV-G and has been tested out on the IBM 360/65 at Educational Testing Service. Double precision is used in floating-point arithmetic throughout the entire program. With minor changes the program should run on any computer with a FORTRAN IV compiler. In computers with a single word length of 36 bits or more, single precision is probably sufficient.

Three versions of the program are available: SIFASP, SFASPL and SFASPF. Their limitations as to the maximum number of populations, variables, factors and independent and nonfixed parameters they can handle as well as their storage requirements on the IBM 360/65 are given in the following table. The given storage requirements assume the programs are overlayed.

	SIFASP	SFASPL	SFASPF
Max. no. of populations (m)	10	10	10
Max. no. of variables (p) before selection	120	200	120
Max. no. of variables (p) after selection	24	40	24
Max. no. of factors (k)	12	20	12
Max. $(\frac{m}{2}\,p(p+1))$	312	820	312
Max. (mpk)	288	800	288
Max. (mp)	48	80	48
Max. $(\frac{m}{2}\,k(k+1))$	78	210	78
Max. no. of independent parameters	120	200	120
Max. no. of nonfixed parameters	150	300	120
Storage requirements (K = 1024 bytes)	144K	280K	146K

SIFASP and SFASPL are identical except for dimensions. Neither of these programs use expressions for second-order derivatives; instead the matrix $E^{(1)}$ of the Fletcher and Powell procedure is an identity matrix (see Fletcher & Powell, 1963; Jöreskog, 1970; or Gruvaeus & Jöreskog,

1970). SFASPF, on the other hand, makes use of such expressions and the speed of convergence is therefore somewhat faster. Standard errors for the estimated parameters can only be obtained with SFASPF.

2.4 Availability

A copy of the program may be obtained by writing to one of the authors. The user must provide a tape on which the program will be loaded. The program will be written on the tape with 80 characters per record. The tape will be unlabeled. The user must specify whether he wants the tape blocked or unblocked, on 7-track or 9-track, in EBCDIC or BCD mode, as well as the density and parity required. Test data will be at the end of the program. The test data are described in the Appendix. Anyone using the program for the first time should make sure that the test data run correctly.

2.5 Disclaimer

Although the program has been working satisfactorily for all data analyzed so far, no claim is made that it is free of error and no warranty is given as to the accuracy and functioning of the program.

3. Input Data

For each data to be analyzed, the input consists of the following.

1. Title card
2. Parameter cards (2)
3. Selection of variables from the input matrix
4. Input matrices
5. Pattern matrices for the parameter matrices
6. Equalities
7. Initial values for the parameter matrices
8. New data set or a STOP card

Sections 3.1 through 3.8 describe in general terms the function and setup of each of the above quantities. Illustrative examples are given in the Appendix.

Whenever a matrix or vector for m populations is read in it is preceded by a format card, containing at most 80 columns, beginning with a left parenthesis and ending with a right parenthesis. The format must specify floating point numbers for the input and parameter matrices, and fixed point numbers for the pattern matrices, consistent with the way in which the elements of the matrix are punched on the following cards. Users who are unfamiliar with FORTRAN are referred to a FORTRAN Manual, where format rules are given. Matrices are punched as one long vector, reading row-wise, each population beginning on a new card. For the symmetric matrices only the lower half of the matrix including the diagonal should be punched.

3.1 Title Card

Whatever appears on this card will appear on the first page of the printed output. All 80 columns of the card are available to the user.

3.2 Parameter Cards (2)

Card 1: All quantities on this card, except for the logical indicators, must be punched as integers right adjusted within the field.

cols. 1-5	Number of populations m
cols. 6-10	Order of the input matrix (p), before selection of variables
cols. 11-15	Number of columns in Λ (k)
cols. 16-25	Total estimated execution time in seconds for all stacked data (SEC). This should be a number slightly less than the time requested on the control cards so the program will have time to print and/or punch results up to that point. (Note: SEC should be read in for each data set and should be the same for all data sets in the stack.)
cols. 31-37	Logical indicators (see below)
cols. 45-46	Integer output indicators (see below)

Logical Indicators (cols. 31-37): The logical indicators control the input and output as described below.

Column 31 determines whether dispersion matrices, or correlation matrices and vectors of standard deviations, are read in as input to determine the matrices to be analyzed.

col. 31: = T , if a dispersion matrix with diagonal is read in for each population

col. 31: = F , if correlation matrices without diagonal, followed by vectors of standard deviations are read in for each population

<u>Column 32</u> determines whether the matrices S_g , $g = 1,2,\dots,m$ to be analyzed are different from the matrices analyzed in the previous data set.

col. 32: = T , if new matrices are to be analyzed (note: this is always true for the first data set)

col. 32: = F , same matrices as for previous data set are analyzed

<u>Column 33</u> determines whether the matrices to be analyzed are scaled or not.

col. 33: = T , matrices to be analyzed are scaled by the program to

$$S_g^* = DS_gD \ , \quad g = 1,2,\dots,m \quad \text{where} \quad D = (\text{diag } S)^{-1/2} \ ,$$

$$S = \frac{1}{n} \sum_{g=1}^{m} n_g S_g \ , \quad n = \sum_{g=1}^{m} n_g$$

col. 33: = F , analysis performed on the unscaled S_g , $g = 1,2,\dots,m$

<u>Column 34</u> determines whether selection of variables from the input matrices is desired.

col. 34: = T , if selection of variables is wanted

col. 34: = F , if <u>no</u> selection of variables is wanted

<u>Column 35</u> determines whether we are considering the standard case or not.

col. 35: = T , the standard case is considered (i.e., Λ_i , $i = 2,3,\dots,m$ are constrained to be equal to Λ_1 ; in this case the

pattern matrix and starting matrix for Λ will be read in for the first population only)

col. 35: = F , we are <u>not</u> considering the standard case (pattern matrices and starting values for all m populations will be read in)

Column 36 determines whether the starting values for Φ_g , $g = 1,2,\ldots,m$ are dispersion matrices or correlation matrices with standard deviations from which the dispersion matrices will be computed.

col. 36: = T , starting Φ's are dispersion matrices

col. 36: = F , starting Φ's are correlation matrices without diagonal and with standard deviations

Column 37 determines whether an accurate or an approximate solution is required.

col. 37: = T , if an approximate solution is required

col. 37: = F , if an accurate solution is required

Integer Output Indicators (cols. 45-46)

Column 45 determines the type of printed output wanted. This can be standard output (S), the matrices to be analyzed and parameter specifications (R), residuals and Σ for each population (C), and technical output from minimization (T).

col. 45: = 0 , for S

col. 45: = 1 , for S + R

col. 45: = 2 , for S + C

col. 45: = 3 , for S + R + C

col. 45: = 4 , for S + T

col. 45: = 5 , for S + R + T

col. 45: = 6 , for S + C + T

col. 45: = 7 , for S + R + C + T

Column 46 determines certain extra printed or punched output. This can be standard errors (F) which is only applicable to SFASPF, punched solution (P), and a scaled solution (G)

col. 46: = 0 , if no extra output is wanted

col. 46: = 1 , for F (never set to 1 for SIFASP or SFASPL)

col. 46: = 2 , for P

col. 46: = 3 , for F + P

col. 46: = 4 , for G

col. 46: = 5 , for F + G

col. 46: = 6 , for P + G

col. 46: = 7 , for F + P + G

Card 2: This card will specify the number of observations or sample size for each population. Thus there will be m integer numbers punched, right-adjusted in five column fields.

Caution: When specifying m , p , k on card 1 of the parameter cards be sure you have read the limitations imposed on them (see 2.3).

3.3 Selection of Variables

These cards will be read in only if the parameter card has a T in column 32 and a T in column 34. Omit otherwise.

The first card will have an integer value p_{new} punched in columns 1-5, right adjusted within the field. This integer will specify the order of the S_g , $g = 1,2,\ldots,m$ after selection $(p_{new} \leq p)$.

The next card will contain integers, right-adjusted in five column fields, (i.e., sixteen such values will fit on one card) specifying which columns (rows) are to be <u>included</u>. For example: if $p = 6$, $p_{new} = 3$ and the 1st, 2nd and 5th columns (rows) are to be <u>excluded</u>. This card would have a 3 punched in column 5, a 4 punched in column 10 and a 6 punched in column 15.

Note that if $p_{new} = p$ there will be no reduction in the size of the S_g but columns (rows) can be interchanged.

3.4 Input Matrices

Omit if column 32 of the parameter card is F . Otherwise read in a format card followed on subsequent cards by the input matrices, starting a new card for each population.

If column 31 of the parameter card is F , the input matrix for the first population, preceded by a format card, is read in without the diagonal. This is immediately followed by a format card and the vector of standard deviations for the first population. Subsequent cards are input matrices without diagonal for the remaining populations each followed on a new card by its vector of standard deviations, and starting a new card for each population. The formats for the first population will apply to subsequent populations.

3.5 Pattern Matrices

The pattern matrices are preceded by a data card with entries in columns 1-3, the column defining the matrix in question, 1 for Λ , 2 for Φ and 3 for ψ .

cols. 1-3: CCC where $C = 0$, if the matrix is fixed

$C = 1$, if the matrix is free

$C = 2$, if the matrix has mixed values

A pattern matrix should be provided only when $C = 2$ (see 2.2).

For example, if columns 1-3 are punched 201, the matrix Λ (i.e., Λ_g, $g = 1,2,\ldots,m$) contains mixed values, Φ (i.e., Φ_g, $g = 1,2,\ldots,m$) is all fixed and ψ (i.e., ψ_g, $g = 1,2,\ldots,m$) is all free. In this case only pattern matrices for Λ_g, $g = 1,2,\ldots,m$ are read in.

The pattern matrix consists of a format card specifying an I-format and subsequent cards with the integer entries of the parameter matrix, beginning a new card for each population.

3.6 Equalities

Omit if the pattern matrices do not contain any elements 2 or 3. Otherwise starting in column 1 punch the four-digit numbers MCCC as described in section 2.2. For each new constrained leader start a new card. The last entry on each "equality" card is a zero indicating more "equality" cards follow, or a four indicating it is the last one (see Appendix for examples).

3.7 Initial Values for the Parameter Matrices

The initial values are preceded by a data card with entries in columns 1-3, the column defining the matrix in question.

cols. 1-3: CCC where $C = 0$, if the matrix is of standard form (see 2.2)

$C = 1$, if the matrix is nonstandard

This card is then followed by the necessary start values (see 2.2) for matrices with $C = 1$. That is, each nonstandard matrix of m populations is read in

with its own format card, starting a new card for each population. If column 36 of the parameter card is F , Φ_1 , preceded by a format card, is read in without the diagonal. This is immediately followed by a format card and the vector of standard deviations for the first population. Subsequent cards are Φ_i , $i = 2,3,\ldots,m$ without diagonal each followed on a new card by its vector of standard deviations, and starting a new card for each population. The formats for the first population will apply to subsequent populations.

3.8 Stacked Data

In sections 3.1 to 3.7 we have described how each set of data should be set up. Any number of such sets of data may be stacked together and analyzed in one run. After the last set of data in the stack, there must be a card with the word STOP punched in columns 1-4.

4. Printed and Punched Output

The output consists of a series of printed and punched tables as described in section 4.1-4.7. Examples of printed output are given in the Appendix.

4.1 Standard Output (S)

The standard output is always obtained, regardless of the value punched in columns 45 and 46 of the parameter card (see 3.2). The standard output consists of the title with parameter listing, the final solution and the result of the test of goodness of fit.

The parameter listing gives the information supplied on the parameter card.

The final solution consists of the three matrices Λ , Φ and ψ , printed for each population.

The test of goodness of fit gives the value of X^2 and the corresponding degrees of freedom. The probability level is also given. This is defined as the probability of getting a X^2 value larger than that actually obtained, given that the hypothesized structure is true.

Just above the table giving the final solution, the following message is printed

"IND = X" .

Usually $X = 0$, but if, for some reason, it has not been possible to determine the final solution, X will be 1, 2, 3, 4 or 5. If IND is 1, 2 or 3, "serious problems" have been encountered and the minimization of the function cannot continue. One reason for this may be erroneous input data. Another

reason may be that a point has been found, where one of the matrices Σ_g is not positive definite. A third reason may be that insufficient arithmetic precision is used. If IND is 4, the number of iterations has exceeded 250. If IND is 5, the time limit SEC has been exceeded (see 3.2). If IND $\neq 0$, the solution obtained so far is automatically punched on cards in such a way as to be immediately available as initial estimates for a new run with the same data. Thus there is little loss of information when execution is terminated with IND $\neq 0$.

4.2 Matrices S_g and Parameter Specifications (R)

If column 45 of the parameter card is 1, 3, 5 or 7 (see 3.2), the matrices to be analyzed, S_g, $g = 1,2,\ldots,m$, as obtained after exclusion of variables and/or scaling (see 1.4), if any, are printed. These matrices are printed row-wise with four decimals. Also a table of parameter specifications, containing the information provided by the pattern matrices (see 2.2), is printed. For each population, three integer matrices are printed corresponding to Λ, Φ and ψ. In each matrix an element is an integer equal to the index of the corresponding parameter in the sequence of independent parameters. Elements corresponding to fixed parameters are 0 and elements corresponding to the same constrained parameter have the same value. Examples are given in the Appendix.

4.3 Technical Output (T)

If column 45 of the parameter card is 4, 5, 6 or 7 (see 3.2), the technical output is printed. This consists of a series of tables which describe the behavior of the iterative procedure and give various measures

of the accuracy of the final solution. Ordinary users will have little interest in these tables.

The first table of the technical output gives the initial estimates for Λ_g , Φ_g , Ψ_g , $g = 1,2,\ldots,m$.

The next two tables show the behavior of the iterative procedure under the steepest descent iterations and under the following iterations by the Fletcher and Powell method. For interpretation of these tables the reader is referred to Gruvaeus and Jöreskog (1970). If something goes wrong, so that IND is 1, 2 or 3 (see 4.1), these tables may contain valuable information.

4.4 Matrices $\hat{\Sigma}_g$ and Residuals (C)

If column 45 of the parameter card is 2, 3, 6 or 7 (see 3.2), the matrices $\hat{\Sigma}_g = \hat{\Lambda}_g\hat{\Phi}_g\hat{\Lambda}'_g + \hat{\Psi}^2_g$ and the residual matrices $S_g - \hat{\Sigma}_g$, $g = 1,2,\ldots,m$, are printed. The matrices $\hat{\Sigma}_g$ are computed from the final solution. If the fit is good, $\hat{\Sigma}_g$ should agree well with S_g and the residual matrices should be small. Elements of the residual matrices may suggest how the hypothesized structure should be modified to obtain a better fit. The matrices are printed row-wise, each element with four decimals.

4.5 Scaled Solution (G)

It column 46 of the parameter card is 4, 5, 6 or 7 (see 3.2), a scaled solution is printed. (See 1.5 on scaling of factors.)

4.6 Standard Errors (F)

If column 46 of the parameter card is 1, 3, 5 or 7 (see 3.2), large sample approximations to the standard errors of the estimated parameters are printed. These are printed row-wise in matrix form and each number is printed with three decimals. The reader is referred to the paper by Jöreskog (1970) for information about how the standard errors are obtained. The standard errors are for the parameters of the unscaled solution.

4.7 Punched Output (P)

If column 46 of the parameter card is 2, 3, 6 or 7 (see 3.2), the final solution is punched on cards. The matrices are punched on cards in vector form, reading row-wise, beginning a new card for each population and each matrix. Each of the three matrices Λ , Φ , ψ are preceded by a format card where by Λ we mean Λ_g , $g = 1,2,\ldots,m$, by Φ we mean Φ_g , $g = 1,2,\ldots,m$ and by ψ we mean ψ_g , $g = 1,2,\ldots,m$. In the standard case only <u>one</u> Λ is punched regardless of the number of populations.

References

Fletcher, R. and Powell, M. J. D. A rapidly convergent descent method for minimization. The Computer Journal, 1963, 6, 163-168.

Gruvaeus, G. and Jöreskog, K. G. A computer program for minimizing a function of several variables. Research Bulletin 70-14. Princeton, N. J.: Educational Testing Service, 1970.

Jöreskog, K. G. Simultaneous factor analysis in several populations. Research Bulletin 70-61. Princeton, N. J.: Educational Testing Service, 1970.

APPENDIX

We shall illustrate how input data are set up and what the printout looks like by means of two small sets of data. These data also serve as test data to be run when the program has been compiled on a different computer.

Both sets of data are analyzed in one run with SFASPF. Pages A4-A5 show card by card how the input data are punched. One line corresponds to one card. Pages A6-A3[illegible] show the corresponding printout obtained.

The first set of data, "Holzinger-Swineford Data," consists of four 9 x 9 correlation matrices without diagonal each with a set of standard deviations. All variables are included in the analysis, and the input matrices are to be scaled by the program before being analyzed. The following model is assumed:

$$\Lambda_1(9 \times 3) = \Lambda_2 = \Lambda_3 = \Lambda_4 = \begin{bmatrix} .798 & 0 & 0 \\ \lambda_4 & 0 & 0 \\ \lambda_7 & 0 & 0 \\ 0 & .796 & 0 \\ 0 & \lambda_{14} & 0 \\ 0 & \lambda_{17} & 0 \\ 0 & 0 & .597 \\ 0 & 0 & \lambda_{24} \\ 0 & 0 & \lambda_{27} \end{bmatrix},$$

$$\Phi_1 = \begin{bmatrix} \phi_1 & & \\ \phi_2 & \phi_3 & \\ \phi_4 & \phi_5 & \phi_6 \end{bmatrix},$$

$$\Phi_2 = \begin{bmatrix} \phi_7 & & \\ \phi_8 & \phi_9 & \\ \phi_{10} & \phi_{11} & \phi_{12} \end{bmatrix} ,$$

$$\Phi_3 = \begin{bmatrix} \phi_{13} & & \\ \phi_{14} & \phi_{15} & \\ \phi_{16} & \phi_{17} & \phi_{18} \end{bmatrix} ,$$

$$\Phi_4 = \begin{bmatrix} \phi_{19} & & \\ \phi_{20} & \phi_{21} & \\ \phi_{22} & \phi_{23} & \phi_{24} \end{bmatrix} ,$$

and the ψ's are constrained to be equal,

$$\Psi_1 = \Psi_2 = \Psi_3 = \Psi_4 = (\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7, \psi_8, \psi_9) \quad .$$

Initial values for Λ_g , Φ_g and Ψ_g were obtained from preliminary analyses of each population separately. All printed output is requested.

The second set of data, "Artificial Data for Illustrative Purposes," consists of two 10 x 10 dispersion matrices with the 10^{th} variable excluded and with the 9^{th} variable moved to the first position. The following model is assumed:

$$\Lambda_1 = \begin{bmatrix} 0 & 0 & \lambda_3 \\ 1 & 0 & 0 \\ \lambda_7 & 0 & 0 \\ \lambda_{10} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \lambda_{17} & 0 \\ 0 & \lambda_{20} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \lambda_{27} \end{bmatrix} , \quad \Lambda_2 = \begin{bmatrix} 0 & 0 & \lambda_{30} \\ 1 & 0 & 0 \\ \lambda_{34} & 0 & 0 \\ \lambda_{37} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \lambda_{44} & 0 \\ 0 & \lambda_{47} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \lambda_{54} \end{bmatrix} ,$$

$$\Phi_1 = \begin{bmatrix} \phi_1 & & \\ \phi_2 & \phi_3 & \\ \phi_4 & \phi_5 & \phi_6 \end{bmatrix} \quad , \quad \Phi_2 = \begin{bmatrix} \phi_7 & & \\ \phi_8 & \phi_9 & \\ \phi_{10} & \phi_{11} & \phi_{12} \end{bmatrix} \quad ,$$

$$\psi_1 = [\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7, \psi_8, \psi_9] \quad ,$$

$$\psi_2 = [\psi_{10}, \psi_{11}, \psi_{12}, \psi_{13}, \psi_{14}, \psi_{15}, \psi_{16}, \psi_{17}, \psi_{18}] \quad .$$

In this analysis we impose the constraints $\lambda_7 = \lambda_{34}$, $\lambda_{10} = \lambda_{37}$, $\lambda_{17} = \lambda_{44}$ $\lambda_{20} = \lambda_{47}$. Initial values have been chosen as 0.9 for all nonfixed λ's except λ_{27} and λ_{54} which have an initial value of 0.4. All ψ's have initial values of 0.8. The Φ_g are read in as correlation matrices, each followed by its vector of standard deviations. Only the standard output, the matrices to be analyzed and the parameter specifications and the standard errors are requested as printed output.

At various places in the output, time estimates are printed. The time shown is the time taken to compute the solution that follows the time estimate. This time includes only the iterations and not the time for printing, except possibly the technical printout.

```
HOLZINGER - SWINEFORD DATA
    4     9     3         220      FTTFTTF        75
   77    79    74    71
(16F5.0)
  .32  .34  .18  .31  .24  .31  .22  .16  .29  .62  .27  .20  .32  .57  .61  .48
  .31  .32  .18  .20  .29  .20  .01  .15  .06  .19  .15  .36  .42  .28  .40  .11
  .07  .18  .35  .44
(16F5.0)
  7.2  4.0  3.0 11.5  4.5  5.5  7.4  4.9  4.7
  .34  .41  .21  .38  .32  .31  .40  .16  .24  .69  .42  .13  .35  .55  .65  .35
  .27  .30  .17  .20  .31  .16  .01  .09  .31  .30  .34  .31  .35  .27  .09  .34
  .27  .27  .38  .38
  6.6  4.8  2.6 11.3  4.7  5.0  6.1  3.9  3.9
  .24  .23  .22  .32  .05  .23  .35  .23  .18  .68  .36  .10  .11  .59  .66  .22
  .01 -.07  .09  .11  .12 -.02 -.01 -.13  .05  .08  .03  .19  .09 -.14 -.06  .16
  .02  .12  .15  .29
  6.7  4.0  2.8  11.  5.2  5.3  7.6  5.2  4.4
  .32  .48  .33  .28  .01  .06  .26  .01  .01  .75  .40  .26  .10  .60  .63  .42
  .32  .22  .15  .07  .36  .12  .05  .03 -.08  .06  .19  .29  .23 -.04  .01 -.05
  .10  .24  .19  .38
  7.4  5.6  2.9 11.8  5.2  5.2  8.8  4.7  4.6
212
(80I1)
000100100000010010000001001
(80I1)
333333333
222222222
222222222
222222222
300130103019302 80
3002301130203029 0
300330123021303 00
300430133022303 10
300530143023303 20
300630153024303 30
300730163025303 40
300830173026303 50
300930183027303 64
111
(5D15.7)
  0.7976910D 00  0.0            0.0            0.3946298D 00  0.0
  0.0            0.4647047D 00  0.0            0.0            0.0
  0.7960486D 00  0.0            0.0            0.8489068D 00  0.0
  0.0            0.7504476D 00  0.0            0.0            0.0
  0.5974687D 00  0.0            0.0            0.4767417D 00  0.0
  0.0            0.5382561D (0
(5D15.7)
  0.1448173D 01  0.4705136D 00  0.1126813D 01  0.6770839D 00  0.2489962D 00
  0.1116507D 01
  0.6523121D 00  0.4931234D 00  0.1037330D 01  0.1060768D 00  0.2084621D 00
  0.7169219D 00
  0.9559079D 00  0.5823390D 00  0.9326544D 00  0.5599713D 00  0.4851271D 00
  0.8452014D 00
  0.8782915D 00  0.4827060D 00  0.8800935D 00  0.9986006D 00  0.3634157D 00
  0.1330246D 01
(5D15.7)
  0.4395305D 00  0.1116168D 01  0.8722642D 00  0.5650579D 00  0.5271286D 00
  0.6637218D 00  0.9594916D 00  0.8604758D 00  0.8953196D 00
  0.4395305D 00  0.1116168D 01  0.8722642D 00  0.5650579D 00  0.5271286D 00
  0.6637218D 00  0.9594916D 00  0.8604758D 00  0.8953196D 00
```

```
  0.4395305D 00  0.1116168D 01  0.8722642D 00  0.5650579D 00  0.5271286D 00
  0.6637218D 00  0.9594916D 00  0.8604758D 00  0.8953196D 00
  0.4395305D 00  0.1116168D 01  0.8722642D 00  0.5650579D 00  0.5271286D 00
  0.6637218D 00  0.9594916D 00  0.8604758D 00  0.8953196D 00
ARTIFICIAL DATA FOR ILLDSTRATIVE PURPOSES
   2   10    3       220      TTFTFFF       11
  61  184
   9
   9    1    2    3    4    5    6    7    8
(5D15.7)
  0.1123817D 01  0.4081763D 00  0.1447772D 01  0.5219276D 00  0.4072726D 00
  0.1052065D 01  0.3072397D 00  0.1245437D-01  0.6370068D-01  0.1071379D 01
  0.2912128D 00  0.1211275D-01  0.1083704D-01  0.8202047D 00  0.1116295D 01
  0.4199384D 00  0.3098141D 00  0.1015779D 00  0.6150361D 00  0.6591857D 00
  0.9807439D 00  0.5187977D 00  0.4486431D 00  0.2629329D 00  0.1809105D 00
  0.8617642D-01  0.4154142D 00  0.1357694D 01  0.1270356D 00  0.6007814D-01
  0.3072833D-01 -0.8269096D-01  0.6330487D-01  0.1879005D 00  0.3374391D 00
  0.9972245D 00  0.2543552D 00 -0.5020824D-01  0.1070006D-01 -0.5398916D-01
  0.1102185D 00  0.2479442D 00  0.2309508D 00  0.3958631D 00  0.1088253D 01
  0.3443672D 00  0.4200382D-01  0.1781234D-01  0.1442567D-01  0.1224889D 00
  0.7856332D 00  0.4556322D 00  0.7753222D 00  0.1233565D 00  0.9885641D 00
  0.9212589D 00  0.1979813D 00  0.7386590D 00  0.2186251D 00  0.1872518D 00
  0.9807595D 00  0.2963624D 00  0.4146429D-01  0.2197816D 00  0.9310318D 00
  0.3549346D 00  0.2088523D 00  0.1883403D 00  0.6932352D 00  0.1116295D 01
  0.3487737D 00  0.8675057D-01  0.1099574D 00  0.5746251D 00  0.7038558D 00
  0.1018827D 01  0.2124934D 00  0.8648761D-02 -0.6976076D-01  0.8738898D-01
  0.1169537D 00  0.1218887D 00  0.1012660D 01 -0.2120913D-01 -0.9495634D-02
 -0.1422416D 00  0.5330332D-01  0.9338591D-01  0.3345596D-01  0.2112455D 00
  0.1220686D 01  0.8619720D-01 -0.1200632D 00 -0.5929148D-01  0.1540501D 00
  0.2108527D-01  0.1208624D 00  0.1506201D 00  0.3197126D 00  0.9956796D 00
  0.1231236D 00  0.4456757D 00  0.5563245D 00  0.2244535D 00  0.1122457D 00
  0.4556788D-01  0.2123452D 00  0.4425136D 00  0.2135788D 00  0.9925635D 00
211
(80I1)
00100030030000000300300000001
00100020020000000200200000001
100710340
101010370
101710440
102010474
111
(40F2.1)
 0 0 91  0 0 9 0 0 9 0 0 01  0 0 9 0 0 9 0 0 01  0 0 4
 0 0 91  0 0 9 0 0 9 0 0 01  0 0 9 0 0 9 0 0 01  0 0 4
(16F5.0)
 .471 .677 .249
(8F10.0)
     1.203      1.062      1.057
 .493 .106 .208
      .807      1.019       .847
(40F2.1)
 8 8 8 8 8 8 8 8 8 8
 8 8 8 8 8 8 8 8 8 8
STOP
```

```
SIMULTANIOUS FACTOR ANALYSIS IN SEVERAL POPULATIONS
HOLZINGER - SWINEFORD DATA
NP(1)=    77
NP(2)=    79
NP(3)=    74
NP(4)=    71
M=     4
P=     9
K=     3
LOGICAL INDICATORS(COLUMNS 51-57):FTTFTTF
OUTPUT INDICATORS= 7 5

ESTIMATED TIME IN SECONDS= 220.
```

POPULATION 1

S

	1	2	3	4	5	6	7	8	9
1	1.066								
2	0.285	0.745							
3	0.373	0.165	1.127						
4	0.323	0.209	0.332	1.018					
5	0.209	0.127	0.283	0.575	0.844				
6	0.292	0.181	0.356	0.602	0.587	1.097			
7	0.489	0.264	0.335	0.179	0.181	0.300	0.974		
8	0.216	0.009	0.166	0.063	0.182	0.164	0.371	1.092	
9	0.463	0.258	0.453	0.118	0.069	0.201	0.369	0.491	1.139

POPULATION 2

S

	1	2	3	4	5	6	7	8	9
1	0.895								
2	0.333	1.073							
3	0.357	0.200	0.846						
4	0.356	0.329	0.283	0.983					
5	0.363	0.159	0.212	0.656	0.920				
6	0.378	0.128	0.307	0.519	0.594	0.906			
7	0.269	0.228	0.225	0.137	0.156	0.240	0.662		
8	0.126	0.009	0.069	0.256	0.239	0.269	0.210	0.692	
9	0.293	0.248	0.073	0.298	0.229	0.228	0.274	0.280	0.784

POPULATION 3

S

	1	2	3	4	5	6	7	8	9
1	0.923								
2	0.199	0.745							
3	0.219	0.188	0.982						
4	0.297	0.042	0.220	0.931					
5	0.357	0.211	0.189	0.696	1.127				
6	0.349	0.087	0.110	0.575	0.707	1.018			
7	0.214	0.009	-0.070	0.088	0.118	0.123	1.027		
8	-0.021	-0.010	-0.143	0.054	0.094	0.034	0.214	1.230	
9	0.086	-0.121	-0.059	0.154	0.021	0.121	0.152	0.321	0.998

POPULATION 4

S

	1	2	3	4	5	6	7	8	9
1	1.126								
2	0.410	1.461							
3	0.523	0.409	1.053						
4	0.308	0.013	0.064	1.072					
5	0.293	0.013	0.011	0.824	1.127				
6	0.420	0.311	0.102	0.615	0.662	0.980			
7	0.523	0.454	0.265	0.182	0.087	0.418	1.377		
8	0.128	0.061	0.031	-0.083	0.064	0.189	0.341	1.004	
9	0.255	-0.050	0.011	-0.054	0.111	0.248	0.233	0.398	1.091

PARAMETER SPECIFICATIONS

POPULATION 1

LAMBDA

0	0	0
1	0	0
2	0	0
0	0	0
0	3	0
0	4	0
0	0	0
0	0	5
0	0	6

PHI

7		
8	9	
10	11	12

PSI

13	14	15	16	17	18	19	20	21

POPULATION 2

LAMBDA

0	0	0
1	0	0
2	0	0
0	0	0
0	3	0
0	4	0
0	0	0
0	0	5
0	0	6

PHI

22		
23	24	
25	26	27

PSI

13	14	15	16	17	18	19	20	21

POPULATION 3

LAMBDA

0	0	0
1	0	0
2	0	0
0	0	0
0	3	0
0	4	0
0	0	0
0	0	5
0	0	6

PHI

28		
29	30	
31	32	33

PSI

13 14 15 16 17 18 19 20 21

POPULATION 4

LAMBDA

0	0	0
1	0	0
2	0	0
0	0	0
0	3	0
0	4	0
0	0	0
0	0	5
0	0	6

PHI

34		
35	36	
37	38	39

PSI

13	14	15	16	17	18	19	20	21

INITIAL SOLUTION

POPULATION 1

LAMBDA

	1	2	3
1	0.798	0.0	0.0
2	0.395	0.0	0.0
3	0.465	0.0	0.0
4	0.0	0.796	0.0
5	0.0	0.849	0.0
6	0.0	0.750	0.0
7	0.0	0.0	0.597
8	0.0	0.0	0.477
9	0.0	0.0	0.538

PHI

	1	2	3
1	1.448		
2	0.471	1.127	
3	0.677	0.249	1.117

PSI

	1	2	3	4	5	6	7	8	9
1	0.440	1.116	0.872	0.565	0.527	0.664	0.959	0.860	0.395

POPULATION 2

LAMBDA

	1	2	3
1	0.798	0.0	0.0
2	0.395	0.0	0.0
3	0.465	0.0	0.0
4	0.0	0.796	0.0
5	0.0	0.849	0.0
6	0.0	0.750	0.0
7	0.0	0.0	0.597
8	0.0	0.0	0.477
9	0.0	0.0	0.538

PHI

	1	2	3
1	0.652		
2	0.493	1.037	
3	0.106	0.208	0.717

PSI

	1	2	3	4	5	6	7	8	9
1	0.440	1.116	0.872	0.565	0.527	0.664	0.959	0.860	0.895

POPULATION 3

LAMBDA

	1	2	3
1	0.798	0.0	0.0
2	0.395	0.0	0.0

3	0.465	0.0	0.0
4	0.0	0.796	0.0
5	0.0	0.849	0.0
6	0.0	0.750	0.0
7	0.0	0.0	0.597
8	0.0	0.0	0.477
9	0.0	0.0	0.538

PHI

	1	2	3
1	0.956		
2	0.582	0.933	
3	0.560	0.485	0.845

PSI

	1	2	3	4	5	6	7	8	9
1	0.440	1.116	0.872	0.565	0.527	0.664	0.959	0.860	0.895

PULATION 4

LAMBDA

	1	2	3
1	0.798	0.0	0.0
2	0.395	0.0	0.0
3	0.465	0.0	0.0
4	0.0	0.796	0.0
5	0.0	0.849	0.0
6	0.0	0.750	0.0
7	0.0	0.0	0.597
8	0.0	0.0	0.477
9	0.0	0.0	0.538

PHI

	1	2	3
1	0.878		
2	0.483	0.880	
3	0.999	0.363	1.330

PSI

	1	2	3	4	5	6	7	8	9
1	0.440	1.116	0.872	0.565	0.527	0.664	0.959	0.860	0.895

BEHAVIOR UNDER STEEPEST DESCENT ITERATIONS

ITER	TRY	ABSCISSA	SLOPE	FUNCTION
1	0	0.0	-0.23365917D 03	0.14549722D 03
	1	0.10000000D 00	-0.13893578D 03	0.12720243D 03
	2	0.24667528D 00	-0.49368280D 02	0.11388437D 03
	3	0.39349876D 00	0.11604644D 02	0.11135238D 03
2	0	0.0	-0.85863194D 02	0.11135238D 03
	1	0.39349876D 00	0.21466636D 03	0.12774018D 03
	2	0.15511569D 00	0.13491058D 01	0.10440717D 03
3	0	0.0	-0.55500031D 02	0.10440717D 03
	1	0.15511569D 00	0.38443661D 02	0.10370996D 03
	2	0.81677801D-01	-0.43149336D-01	0.10224375D 03
4	0	0.0	-0.43852885D 02	0.10224375D 03
	1	0.81677801D-01	-0.15806099D 01	0.10031534D 03

BEHAVIOR UNDER FLEPOW ITERATIONS

ITER	TRY	ABSCISSA	SLOPE	FUNCTION
1	0	0.0	-0.27145608D 02	0.10031534D 03
	1	0.10000000D 00	-0.24345003D 02	0.97741812D 02
	2	0.13069517D 01	0.21946265D 02	0.90882199D 02
	3	0.89284712D 00	-0.10321629D 01	0.87133319D 02
2	0	0.0	-0.44337646D 01	0.87133319D 02
	1	0.89284712D 00	0.23624045D 02	0.91578710D 02
	2	0.35146570D 00	0.68090146D 00	0.86359070D 02
	3	0.31668022D 00	-0.18325592D-01	0.86347700D 02
3	0	0.0	-0.37850761D 01	0.86347700D 02
	1	0.31668022D 00	0.87552759D 01	0.87172461D 02
	2	0.91861742D-01	0.44427133D-01	0.86178250D 02
4	0	0.0	-0.17366576D 01	0.86178250D 02
	1	0.91861742D-01	0.15386002D 00	0.86105439D 02
5	0	0.0	-0.51584948D 00	0.86105439D 02
	1	0.91861742D-01	0.12009728D 01	0.86135550D 02
	2	0.28619189D-01	0.26374131D-03	0.86098023D 02
6	0	0.0	-0.37455577D 00	0.86098023D 02
	1	0.28619189D-01	0.39157427D 00	0.86098293D 02
	2	0.13939770D-01	0.13696321D-05	0.86095416D 02
7	0	0.0	-0.23740794D 00	0.86095416D 02
	1	0.13939770D-01	-0.12891634D-02	0.86093752D 02
8	0	0.0	-0.15794796D 00	0.86093752D 02
	1	0.13939770D-01	-0.38015644D-01	0.86092387D 02
	2	0.18375661D-01	0.38591097D-05	0.86092302D 02
9	0	0.0	-0.21499105D 00	0.86092302D 02
	1	0.18375661D-01	0.24735026D-01	0.86090552D 02
	2	0.16485781D-01	-0.69778622D-06	0.86090528D 02
10	0	0.0	-0.49997638D 00	0.86090528D 02
	1	0.16485781D-01	-0.20631646D 00	0.86084703D 02
	2	0.27979363D-01	0.27431202D-04	0.86083516D 02
11	0	0.0	-0.72608318D 00	0.86083516D 02
	1	0.27979363D-01	-0.75950048D-01	0.86072267D 02
	2	0.31213287D-01	0.11628020D-04	0.86072144D 02
12	0	0.0	-0.36240696D 00	0.86072144D 02
	1	0.31213287D-01	0.48471324D 00	0.86074044D 02
	2	0.13367970D-01	0.50631263D-05	0.86069721D 02
13	0	0.0	-0.14453614D 00	0.86069721D 02
	1	0.13367970D-01	0.21331617D 00	0.86070179D 02
	2	0.54061048D-02	0.47653989D-06	0.86069330D 02
14	0	0.0	-0.43866881D-01	0.86069330D 02
	1	0.54061048D-02	0.11994020D 00	0.86069536D 02
	2	0.14501964D-02	0.84358848D-07	0.86069298D 02
15	0	0.0	-0.15625268D-01	0.86069298D 02

	1	0.14501964D-02	0.18417496D-01	0.86069300D 02
	2	0.66530294D-03	0.15248343D-09	0.86069293D 02
16	0	0.0	-0.13203627D-01	0.86069293D 02
	1	0.66530294D-03	0.14963924D-02	0.86069289D 02
	2	0.59757824D-03	0.18702463D-07	0.86069289D 02
17	0	0.0	-0.33693271D-01	0.86069289D 02
	1	0.59757824D-03	-0.18933627D-01	0.86069274D 02
	2	0.13641498D-02	0.18559817D-05	0.86069266D 02
18	0	0.0	-0.29932166D-01	0.86069266D 02
	1	0.13641498D-02	-0.66029722D-03	0.86069245D 02
19	0	0.0	-0.19331608D-01	0.86069245D 02
	1	0.13641498D-02	0.11741652D 00	0.86069312D 02
	2	0.19287031D-03	0.65644734D-08	0.86069244D 02
20	0	0.0	-0.19779190D-02	0.86069244D 02
	1	0.19287031D-03	0.21083499D-02	0.86069244D 02
	2	0.93357010D-04	-0.16082493D-07	0.86069243D 02
21	0	0.0	-0.29334193D-02	0.86069243D 02
	1	0.93357010D-04	-0.16759904D-03	0.86069243D 02
22	0	0.0	-0.68556727D-02	0.86069243D 02
	1	0.93357010D-04	-0.42433654D-02	0.86069243D 02
	2	0.24500376D-03	-0.22216035D-06	0.86069242D 02
23	0	0.0	-0.32632037D-02	0.86069242D 02
	1	0.24500376D-03	0.77309223D-02	0.86069243D 02
	2	0.72720393D-04	0.47837506D-07	0.86069242D 02

TIME= 57.37

MAXIMUM LIKELIHOOD SOLUTION

IND= 0

POPULATION 1

LAMBDA

	1	2	3
1	0.798	0.0	0.0
2	0.473	0.0	0.0
3	0.568	0.0	0.0
4	0.0	0.796	0.0
5	0.0	0.847	0.0
6	0.0	0.746	0.0
7	0.0	0.0	0.597
8	0.0	0.0	0.496
9	0.0	0.0	0.578

PHI

	1	2	3
1	0.838		
2	0.475	0.911	
3	0.908	0.348	1.238

PSI

	1	2	3	4	5	6	7	8	9
1	0.693	0.904	0.858	0.604	0.530	0.665	0.812	0.875	0.825

POPULATION 2

LAMBDA

	1	2	3
1	0.798	0.0	0.0
2	0.473	0.0	0.0
3	0.568	0.0	0.0
4	0.0	0.796	0.0
5	0.0	0.847	0.0
6	0.0	0.746	0.0
7	0.0	0.0	0.597
8	0.0	0.0	0.496
9	0.0	0.0	0.578

PHI

	1	2	3
1	0.731		
2	0.562	0.928	
3	0.522	0.486	0.557

PSI

	1	2	3	4	5	6	7	8	9
1	0.693	0.904	0.858	0.604	0.530	0.665	0.812	0.875	0.825

POPULATION 3

LAMBDA

	1	2	3

1	0.798	0.0	0.0
2	0.473	0.0	0.0
3	0.568	0.0	0.0
4	0.0	0.796	0.0
5	0.0	0.847	0.0
6	0.0	0.746	0.0
7	0.0	0.0	0.597
8	0.0	0.0	0.496
9	0.0	0.0	0.578

PHI

	1	2	3
1	0.592		
2	0.469	1.059	
3	0.074	0.199	0.853

PSI

	1	2	3	4	5	6	7	8	9
1	0.693	0.904	0.858	0.604	0.530	0.665	0.812	0.875	0.825

ON 4

LAMBDA

	1	2	3
1	0.798	0.0	0.0
2	0.473	0.0	0.0
3	0.568	0.0	0.0
4	0.0	0.796	0.0
5	0.0	0.847	0.0
6	0.0	0.746	0.0
7	0.0	0.0	0.597
8	0.0	0.0	0.496
9	0.0	0.0	0.578

PHI

	1	2	3
1	1.128		
2	0.386	1.127	
3	0.627	0.265	1.191

PSI

	1	2	3	4	5	6	7	8	9
1	0.693	0.904	0.858	0.604	0.530	0.665	0.812	0.875	0.825

TEST OF GOODNESS OF FIT

CHISQUARE WITH 141 DEGREES OF FREEDOM IS 172.1385

PROBABILITY LEVEL IS 0.038

POPULATION 1

SIGMA = LAMBDA*PHI*LAMBDA'+PSI**2

	1	2	3	4	5	6	7	8	9
1	1.013								
2	0.316	1.004							
3	0.379	0.225	1.006						
4	0.302	0.179	0.215	0.942					
5	0.321	0.191	0.229	0.614	0.934				
6	0.283	0.168	0.201	0.541	0.575	0.949			
7	0.433	0.257	0.308	0.166	0.176	0.155	1.10[illegible]		
8	0.359	0.213	0.256	0.137	0.146	0.129	0.367	1.070	
9	0.419	0.249	0.298	0.160	0.171	0.150	0.428	0.355	1.[illegible]

RESIDUALS = SIGMA-S

	1	2	3	4	5	6	7	8	9
1	-0.053								
2	0.031	0.259							
3	0.007	0.060	-0.120						
4	-0.021	-0.030	-0.117	-0.076					
5	0.113	0.064	-0.054	0.040	0.090				
6	-0.009	-0.013	-0.155	-0.061	-0.011	-0.148			
7	-0.056	-0.007	-0.027	-0.014	-0.005	-0.145	0.128		
8	0.143	0.204	0.089	0.074	-0.036	-0.035	-0.005	-0.022	
9	-0.044	-0.009	-0.155	0.042	0.102	-0.051	0.059	-0.136	-0.[illegible]

POPULATION 2

SIGMA = LAMBDA*PHI*LAMBDA'+PSI**2

	1	2	3	4	5	6	7	8	9
1	0.945								
2	0.276	0.981							
3	0.331	0.196	0.972						
4	0.357	0.212	0.254	0.953					
5	0.380	0.226	0.270	0.626	0.946				
6	0.334	0.199	0.238	0.551	0.586	0.958			
7	0.249	0.148	0.177	0.231	0.246	0.217	0.858		
8	0.206	0.122	0.147	0.192	0.204	0.180	0.165	0.903	
9	0.241	0.143	0.171	0.224	0.238	0.210	0.192	0.160	0.867

RESIDUALS = SIGMA-S

	1	2	3	4	5	6	7	8	9
1	0.049								
2	-0.057	-0.093							
3	-0.026	-0.004	0.126						
4	0.001	-0.117	-0.029	-0.030					
5	0.017	0.067	0.059	-0.030	0.026				
6	-0.044	0.070	-0.069	0.032	-0.007	0.052			
7	-0.021	-0.080	-0.048	0.094	0.090	-0.023	0.197		
8	0.080	0.114	0.078	-0.064	-0.035	-0.089	-0.045	0.211	
9	-0.0[illegible]3	-0.105	0.098	-0.075	0.009	-0.018	-0.081	-0.120	0.083

ATION 3

SIGMA = LAMBDA*PHI*LAMBDA'+PSI**2

	1	2	3	4	5	6	7	8	9
1	0.856								
2	0.224	0.949							
3	0.268	0.159	0.927						
4	0.298	0.177	0.212	1.036					
5	0.317	0.188	0.225	0.714	1.041				
6	0.279	0.166	0.198	0.629	0.669	1.031			
7	0.035	0.021	0.025	0.095	0.101	0.089	0.964		
8	0.029	0.017	0.021	0.079	0.084	0.074	0.253	0.975	
9	0.034	0.020	0.024	0.092	0.097	0.086	0.295	0.245	0.966

RESIDUALS = SIGMA-S

	1	2	3	4	5	6	7	8	9
1	-0.066								
2	0.025	0.204							
3	0.049	-0.029	-0.054						
4	0.001	0.135	-0.008	0.105					
5	-0.040	-0.023	0.036	0.018	-0.086				
6	-0.070	0.078	0.088	0.054	-0.038	0.013			
7	-0.179	0.012	0.095	0.007	-0.018	-0.034	-0.063		
8	0.051	0.027	0.164	0.025	-0.011	0.040	0.039	-0.254	
9	-0.052	0.141	0.084	-0.063	0.076	-0.035	0.143	-0.077	-0.032

POPULATION 4

SIGMA = LAMBDA*PHI*LAMBDA'+PSI**2

	1	2	3	4	5	6	7	8	9
1	1.197								
2	0.426	1.070							
3	0.511	0.303	1.100						
4	0.245	0.146	0.175	1.079					
5	0.261	0.155	0.186	0.760	1.089				
6	0.230	0.136	0.163	0.669	0.712	1.069			
7	0.299	0.177	0.213	0.126	0.134	0.118	1.085		
8	0.248	0.147	0.176	0.105	0.111	0.098	0.353	1.058	
9	0.289	0.172	0.206	0.122	0.130	0.114	0.411	0.341	1.[illegible]

RESIDUALS = SIGMA-S

	1	2	3	4	5	6	7	8	9
1	0.072								
2	0.016	-0.391							
3	-0.012	-0.106	0.047						
4	-0.062	0.133	0.111	0.007					
5	-0.032	0.142	0.175	-0.064	-0.038				
6	-0.190	-0.175	0.062	0.054	0.050	0.089			
7	-0.224	-0.277	-0.052	-0.056	0.047	-0.300	-0.292		
8	0.120	0.087	0.145	0.188	0.047	-0.091	0.012	[illegible].054	
9	0.034	0.222	0.195	0.176	0.019	-0.134	0.178	[illegible].057	-0.[illegible]

SCALED SOLUTION

POPULATION 1

LAMBDA

	1	2	3
1	0.721	0.0	0.0
2	0.428	0.0	0.0
3	0.513	0.0	0.0
4	0.0	0.797	0.0
5	0.0	0.848	0.0
6	0.0	0.747	0.0
7	0.0	0.0	0.583
8	0.0	0.0	0.484
9	0.0	0.0	0.565

PHI

	1	2	3
1	1.024		
2	0.52[illegible]	0.908	
3	1.0	0.356	1.298

PSI

	1	2	3	4	5	6	7	8	9
1	0.693	0.904	0.858	0.604	0.530	0.665	0.812	0.875	0.825

POPULATION 2

LAMBDA

	1	2	3
1	0.721	0.0	0.0
2	0.428	0.0	0.0
3	0.513	0.0	0.0
4	0.0	0.797	0.0
5	0.0	0.848	0.0
6	0.0	0.747	0.0
7	0.0	0.0	0.583
8	0.0	0.0	0.484
9	0.0	0.0	0.565

PHI

	1	2	3
1	0.894		
2	0.621	0.925	
3	0.591	0.497	0.584

PSI

	1	2	3	4	5	6	7	8	9
1	0.693	0.904	0.858	0.604	0.530	0.665	0.812	0.875	0.825

POPULATION 3

LAMBDA

	1	2	3
1	0.721	0.0	0.0
2	0.428	0.0	0.0

	1	2	3
3	0.513	0.0	0.0
4	0.0	0.797	0.0
5	0.0	0.848	0.0
6	0.0	0.747	0.0
7	0.0	0.0	0.583
8	0.0	0.0	0.484
9	0.0	0.0	0.565

PHI

	1	2	3
1	0.724		
2	0.518	1.056	
3	0.084	0.203	0.895

PSI

	1	2	3	4	5	6	7	8	9
1	0.693	0.904	0.858	0.604	0.530	0.665	0.812	0.875	0.825

LATION 4

LAMBDA

	1	2	3
1	0.721	0.0	0.0
2	0.428	0.0	0.0
3	0.513	0.0	0.0
4	0.0	0.797	0.0
5	0.0	0.848	0.0
6	0.0	0.747	0.0
7	0.0	0.0	0.583
8	0.0	0.0	0.484
9	0.0	0.0	0.565

PHI

	1	2	3
1	1.380		
2	0.426	1.124	
3	0.710	0.271	1.249

PSI

	1	2	3	4	5	6	7	8	9
1	0.693	0.904	0.858	0.604	0.530	0.665	0.812	0.875	0.825

STANDARD ERRORS

POPULATION 1

LAMBDA

	1	2	3
1	0.0	0.0	0.0
2	0.C83	0.0	0.0
3	0.087	0.0	0.0
4	0.0	0.C	0.0
5	0.0	0.062	0.0
6	C.0	0.059	0.0
7	0.0	0.0	0.C
8	0.0	0.0	0.091
9	0.0	0.C	0.098

PHI

	1	2	3
1	0.242		
2	0.152	0.195	
3	0.232	0.177	0.396

PSI

	1	2	3	4	5	6	7	8	9
1	0.055	0.041	0.042	0.037	0.042	0.036	0.047	0.043	0.046

POPULATION 2

LAMBDA

	1	2	3
1	0.0	0.0	0.0
2	0.083	0.0	0.0
3	0.C87	0.0	0.0
4	0.0	0.0	0.0
5	0.0	0.062	0.0
6	0.0	0.059	0.0
7	0.0	0.C	0.0
8	0.0	0.0	0.091
9	0.0	0.0	0.098

PHI

	1	2	3
1	0.220		
2	0.151	0.196	
3	0.163	0.154	0.237

PSI

	1	2	3	4	5	6	7	8	9
1	0.055	0.041	0.042	0.037	0.042	0.036	0.047	0.043	0.046

POPULATION 3

LAMBDA

	1	2	3
1	0.0	0.0	0.0
2	0.C83	0.0	0.0

	1	2	3
3	0.037	0.0	0.0
4	0.0	0.0	0.0
5	0.0	0.062	0.0
6	0.0	0.059	0.0
7	0.0	0.0	0.0
8	0.0	0.0	0.091
9	0.0	0.0	0.098

PHI

	1	2	3
1	0.200		
2	0.149	0.226	
3	0.154	0.167	0.311

PSI

	1	2	3	4	5	6	7	8	9
1	0.055	0.041	0.042	0.037	0.042	0.0[illegible]6	0.047	0.043	0.046

ULATION 4

LAMBDA

	1	2	3
1	0.0	0.0	0.0
2	0.083	0.0	0.0
3	0.08[illegible]	0.0	0.0
4	0.0	0.0	0.0
5	0.0	0.062	0.0
6	0.0	0.059	0.0
7	0.0	0.0	0.0
8	0.0	0.0	0.091
9	0.0	0.0	0.098

PHI

	1	2	3
1	0.305		
2	0.182	0.243	
3	0.233	0.194	0.395

PSI

	1	2	3	4	5	6	7	8	9
1	0.055	0.041	0.042	0.037	0.042	0.036	0.047	0.043	0.046

```
SIMULTANIOUS FACTOR ANALYSIS IN SEVERAL POPULATIONS
ARTIFICIAL DATA FOR ILLUSTRATIVE PURPOSES
NP(1)=   61
NP(2)=  184
M=    2
P=    10
K=    3
LOGICAL INDICATORS(COLUMNS 51-57):TTFTFFF
OUTPUT INDICATORS= 1 1

ESTIMATED TIME IN SECONDS= 220.
```

ULATION 1

S

	1	2	3	4	5	6	7	8	9
1	1.088								
2	0.254	1.124							
3	-0.050	0.408	1.448						
4	0.011	0.522	0.407	1.052					
5	-0.054	0.307	0.012	0.064	1.071				
6	0.110	0.291	0.013	0.011	0.820	1.116			
7	0.248	0.420	0.310	0.102	0.615	0.659	0.981		
8	0.231	0.519	0.449	0.263	0.181	0.086	0.415	1.358	
9	0.396	0.127	0.060	0.031	-0.083	0.063	0.188	0.337	0.997

ULATION 2

S

	1	2	3	4	5	6	7	8	9
1	0.996								
2	0.086	0.921							
3	-0.120	0.198	0.739						
4	-0.059	0.219	0.187	0.981					
5	0.154	0.296	0.041	0.220	0.931				
6	0.021	0.355	0.209	0.188	0.693	1.116			
7	0.121	0.349	0.087	0.110	0.575	0.704	1.019		
8	0.151	0.212	0.009	-0.070	0.087	0.117	0.122	1.013	
9	0.320	-0.021	-0.009	-0.142	0.053	0.093	0.033	0.211	1.221

```
PARAMETER SPECIFICATIONS

     POPULATION 1

LAMBDA
    0    0    1
    0    0    0
    2    0    0
    3    0    0
    0    0    0
    0    4    0
    0    5    0
    0    0    0
    0    0    6

PHI
    7
    8    9
   10   11   12

PSI
   13   14   15   16   17   18   19   20   21

     POPULATION 2

LAMBDA
    0    0   22
    0    0    0
    2    0    0
    3    0    0
    0    0    0
    0    4    0
    0    5    0
    0    0    0
    0    0   23

PHI
   24
   25   26
   27   28   29

PSI
   30   31   32   33   34   35   36   37   38

TIME=    58.91
```

MAXIMUM LIKELIHOOD SOLUTION

IND= 0

PULATION 1

LAMBDA

	1	2	3
1	0.0	0.0	0.485
2	[illegible]	0.0	0.0
3	[illegible]	0.0	0.0
4	0.584	[illegible]	0.0
5	0.0	1.000	0.0
[illegible]	0.0	1.174	0.0
7	0.0	0.96[illegible]	0.0
8	0.0	0.0	1.000
9	0.0	0.0	0.510

PHI

	1	2	3
1	0.902		
2	0.276	0.647	
3	0.473	0.159	0.669

PSI

	1	2	3	4	5	6	7	8	9
1	0.965	0.466	1.110	0.870	0.584	0.496	0.670	0.830	0.90

PULATION 2

LAMBDA

	1	2	3
1	0.0	0.0	1.527
2	1.000	0.0	0.0
3	0.495	0.0	0.0
4	0.584	0.0	0.0
5	0.0	1.000	0.0
6	0.0	1.174	0.0
7	0.0	0.962	0.0
8	0.0	0.0	1.000
9	0.0	0.0	1.984

PHI

	1	2	3
1	0.408		
2	0.300	0.604	
3	0.004	0.047	0.105

PSI

	1	2	3	4	5	6	7	8	9
1	0.867	0.719	0.797	0.915	0.596	0.526	0.662	0.953	0.89

TEST OF GOODNESS OF FIT

CHISQUARE WITH 52 DEGREES OF FREEDOM IS 88.7335

PROBABILITY LEVEL IS 0.001

STANDARD ERRORS

POPULATION 1

LAMBDA

	1	2	3
1	0.0	0.0	0.247
2	0.0	0.0	0.0
3	0.123	0.0	0.0
4	0.132	0.0	0.0
5	0.0	0.0	0.0
6	0.0	0.091	0.0
7	0.0	0.082	0.0
8	0.0	0.0	0.0
9	0.0	0.0	0.245

PHI

	1	2	3
1	0.270		
2	0.123	0.151	
3	0.167	0.123	0.348

PSI

	1	2	3	4	5	6	7	8	9
1	0.098	0.214	0.105	0.090	0.075	0.093	0.076	0.182	0.09

POPULATION 2

LAMBDA

	1	2	3
1	0.0	0.0	0.645
2	0.0	0.0	0.0
3	0.123	0.0	0.0
4	0.132	0.0	0.0
5	0.0	0.0	0.0
6	0.0	0.091	0.0
7	0.0	0.082	0.0
8	0.0	0.0	0.0
9	0.0	0.0	0.905

PHI

	1	2	3
1	0.120		
2	0.063	0.095	
3	0.030	0.033	0.071

PSI

	1	2	3	4	5	6	7	8	9
1	0.072	0.119	0.033	0.055	0.046	0.059	0.044	0.065	0.11